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D-brane chemistry

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Abstract: We study several different kinds of bound states built from D-branes and orientifolds. These states are to atoms what branonium — the bound state of a brane and its anti-brane — is to positronium, inasmuch as they typically involve a light brane bound to a much heavier object with conserved charges which forbid the system's decay. We find the fully relativistic motion of a probe Dp'-brane in the presence of source Dp-branes is integrable by quadratures. Keplerian conic sections are obtained for special choices for p and p' and the systems are shown to be equivalent to non-relativistic systems. Their quantum behaviour is also equivalent to the corresponding non-relativistic limit. In particular the $p = 6$, $p' = 0$ case is equivalent to a non-relativistic dyon in a magnetic monopole background, with trajectories in the surface of a cone. We also show that the motion of probe branes about D6-branes in IIA theory is equivalent to the motion of the corresponding probes in the uplift to M theory in 11 dimensions, for which there are no D6-branes but their fields are replaced by a particular Taub-NUT geometry. We further discuss the interactions of D-branes and orientifold planes having the same dimension. This system behaves at large distances as a brane-brane system but at shorter distances it does not have the tachyon instability.

Keywords: Superstrings and Heterotic Strings, D-branes, Cosmology of Theories beyond the SM.

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1. Introduction

We consider the motion of Dp'-branes moving in the presence of sources, which we take to be either Dp-branes having $p > p'$ or Op (orientifold) planes having $p = p'$. These systems can contain the brane analogs of atoms, inasmuch as they can form stable orbits which are stable against annihilation or decay by virtue of the conserved charges which their constituents carry. (This represents an important difference between these systems and the branonium systems — i.e. a Dp-brane and $\overline{\mathrm{D}}p$-antibrane — studied in [1, 2, 3].)

We find that the Dp-Dp' systems experience an attractive interaction precisely for those choices of $p - p'$ which correspond to the existence of a tachyon in the open-string spectrum, which presumably therefore indicates the instability toward the formation of a supersymmetric bound state built from the Dp-Dp' pair. We compute the classical trajectories (and some quantum properties) for the motion of the Dp'-brane in the probe-brane limit. For some choices the resulting orbits are simply the usual conic sections of the non-relativistic Kepler problem, even though they apply to the fully relativistic motion of the probe brane. Our study of these systems is motivated by the remarkable properties of

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the related branonium systems, for which the fully relativistic motion of a probe antibrane through the fields sourced by a stack of source branes may be completely reduced to quadratures. We find this property to be shared by the broader class of systems consisting of Dp'-branes orbiting Dp-brane sources.

Our presentation is organized as follows. Section 2, immediately following, sets up and solves the equations of motion for the probe Dp'-brane system in the presence of N source Dp-branes. Section 3 then examines how this motion looks for the special situation of IIA theory where the source branes are D6 branes. In this case the theory may be uplifted to M-theory in 11 dimensions with the D6 branes disappearing, but with the fields they source being replaced by an extended Taub-NUT geometry. We show in this section that the motion of probe branes about D6-branes in IIA theory is equivalent to the motion of equivalent probes in the Taub-NUT space. Section 4 then examines several features of the interaction potential between Dp-branes and $Op^{\pm}$ orientifold planes. We close with our conclusions in section 5.

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2. Brane atoms: Dp'-branes in Dp-backgrounds

In this section we generalize the results of [1] to the situation where the dimension, p', of the probe brane is different from the dimension, p, of the source brane (see figure 1). We consider the higher-dimensional Dp-brane (or set of Dp-branes) as the source of a background field configuration within which the probe Dp'-brane moves. This is a good approximation when the mass of the higher-dimensional brane (or set of branes) is bigger than the mass of the lower-dimensional one. In particular, it is always the case when the directions transverse to the lower-dimensional brane but parallel to the higher-dimensional one are very large or non-compact, and we henceforth assume this to be true for the remainder of this section.

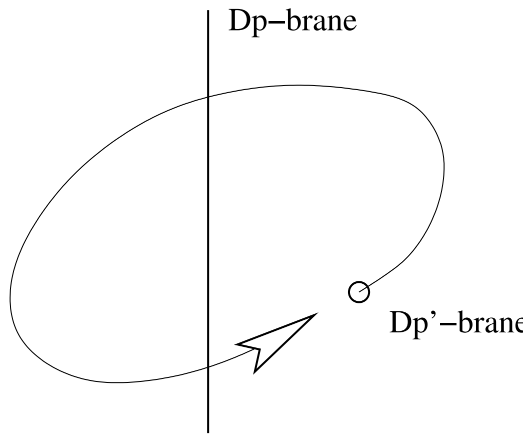

Figure 1: A Dp'-brane orbits a higher dimensional Dp-brane.

2.1 Constituents

We begin by reviewing the relevant properties of the background fields and the probe branes.

The background. We consider the Dp-brane to be static, acting only as a source for a supergravity background. This means that we ignore the massive closed-string modes which could also have been sourced, and so implies the validity of a low-energy expansion (i.e. we neglect α' corrections). Since these modes become relevant at short distances compared with the string scale, $r \ll l_s$, at these distances the supergravity framework breaks down and the system is better described within the open-string picture.

The background sourced by the higher-dimensional Dp-brane is given by the following solution to the bosonic field equations of ten-dimensional supergravity

$$\begin{aligned} ds^2 &= h^{-1/2}(-dt^2 + dx_1^2 + \cdots + dx_p^2) + h^{1/2}(dy_{p+1} + \cdots + dy_9) \\ C_{0\cdots p} &= 1 - \frac{1}{h} \qquad \text{and} \qquad e^{\phi} = h^{(3-p)/4} \end{aligned} \tag{2.1}$$

where $C_{(p+1)}$ is the $(p+1)$-dimensional Ramond-Ramond (RR) form which couples to brane charge, and ϕ is the dilaton. When $p < 7$ the function h is given explicitly by

$$h = 1 + \frac{k}{r^{7-p}} \tag{2.2}$$

with $r^2 = y_{p+1}^2 + \cdots + y_9^2$ and $k = c_p\, g_s N\, l_s^{7-p}$, where $c_p = (2\sqrt{\pi})^{5-p}\,\Gamma(\frac{7-p}{2})$.

Open-string tachyons and stability. As mentioned above, we take the Dp'-brane to be a probe which moves in the above background, created by the higher-dimensional Dp-brane. Although this description works well for large brane separations, it fails at distances of the order of l_s, where there can be open string modes that become massless or can become tachyonic. We now analyse the stability of the system from this open-string point of view, which we shall see nicely complements our later analysis of the equation of motion of the probe brane as it moves through the background fields.

In the brane-antibrane case, an open-string tachyon appears once the branes are separated by $r \sim l_s$, signalling the system's instability towards brane-antibrane annihilation. A similar tachyon develops for Dp'-branes orbiting Dp-branes if $|p' - p| < 4$. If $|p' - p| = 4$ the would-be tachyon mode is instead a massless scalar mode and the static system is supersymmetric in the sense that the brane system preserves 1/4 of the bulk supersymmetries. (This supersymmetry and flat direction is reflected in the absence of a potential of interaction between the branes when they are parallel and are not in relative motion.) When $|p' - p| > 4$ the lowest open-string mode at short distances is massive.

As we shall see, the existence of this tachyonic open-string mode at short distances correlates with whether the one-loop brane interaction potentials are attractive — a result which further generalizes to the more generic case of branes at angles or branes with magnetic fluxes. There is a generic rule[1] concerning these type of systems: attractive (repulsive) potentials appear when the lowest open-string mode at short distances is tachyonic (massive). The static (supersymmetric) cases correspond to the lowest mode being massless. This correlation has a simple physical interpretation: the existence of an open-string tachyon signals an instability towards the formation of a bound state of the two branes through the attractive interaction potential, leading to a lower-energy (i.e. tension) configuration, which is typically supersymmetric and stable (and presumably represents the global energy minimum). One would expect from this interpretation that the two branes are attracted to one another in order to reach this minimum, and this is what we shall find.

[1] This rule was observed a long time ago, see for instance [7].

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The action. In the background (2.1), we introduce a probe Dp'-brane, whose action is given by a Born-Infeld (BI) and Wess-Zumino (WZ) term, according to

$$S = -T_{p'} \int d^{p'+1}\xi \, e^{-\phi} \sqrt{|g_{AB}\partial_\alpha X^A \partial_\beta X^B|} - q\, T_{p'} \int C_{p'}$$
$$\alpha, \beta = 0, \ldots, p'\,; \qquad A, B = 0, \ldots, 9\,, \tag{2.3}$$

where $X^A(\xi_\alpha)$ represents the embedding of the brane into spacetime, and $|M_{\alpha\beta}|$ denotes the absolute value of the determinant of any matrix $M_{\alpha\beta}$. In this expression the world-volume gauge field $F_{\alpha\beta}(\xi_\alpha)$ has been turned off, $T_{p'}$ is the tension of the probe brane and $q = \pm 1$ the corresponding RR charge.

After gauge-fixing the reparameterization invariance of the action (2.3) using the so-called static gauge, $X^\alpha(\xi^\beta) = \xi^\alpha$, we assume the brane moves rigidly (i.e. $X^a = X^a(t)$) which means that we neglect any oscillatory modes of the brane.[2] The probe brane is also imagined to be oriented parallel to some of the dimensions of the higher-dimensional source brane during this rigid motion. Under these circumstances the probe brane moves like a particle in the $9 - p$ dimensions transverse to the source brane. The BI part of the action then becomes

$$S_{BI} = -m \int dt\, h^{(p-3)/4} \sqrt{\left| g_{\alpha\beta} + \delta^0_\alpha \delta^0_\beta \dot{X}_a \dot{X}^a \right|} \qquad a = p' + 1, \ldots, 9\,, \tag{2.4}$$

where $g_{\alpha\beta} = h^{-1/2}\delta_{\alpha\beta}$. Since our interest is in the case $p' < p$, we can assume the directions parallel to the probe brane are toroidally compactified without any need to modify the background field configurations. With this understanding, the factor m in this last equation is the 'mass' of the Dp'-brane $m = T_{p'} V_{p'}$ with $V_{p'}$ the volume of these compact dimensions.

From here on we reserve the coordinates y^n (with $n = p+1, \ldots, 9$) for the dimensions transverse to the Dp-brane, and the coordinates x^i (with $i = p' + 1, \ldots, p$) for those dimensions transverse to the Dp'-brane, but parallel to the Dp-brane. With this choice we have $\dot{X}_a \dot{X}^a = h^{-1/2}\dot{x}_i^2 + h^{1/2}\dot{y}_n^2$. Evaluating the determinant finally allows the BI part of the action to take the general form

$$S_{BI} = -m \int dt\, h^{(p-p'-4)/4} \sqrt{1 - \dot{x}_i^2 - h\, \dot{y}_n^2}$$
$$i = p' + 1, \ldots, p\,; \qquad n = p + 1, \ldots, 9\,. \tag{2.5}$$

The WZ term can differ from zero in any of the following three circumstances: (i) If $p = p'$; (ii) If the Dp-brane carries a lower-dimensional charge (e.g. a bound state of the Dp-brane with lower-dimensional branes); or (iii) If there are several probe branes and a Myers-type of effect develops [16], etc. In this section we consider Dp-branes without magnetic flux and we take only a single probe. Since $p \neq p'$ and the world-volume gauge fields on the probe brane are set to zero, the Ramond-Ramond fields do not affect the movement of the probe brane. Therefore, generically at long distances the probe brane only feels the influence of gravity and the dilaton field.

[2] These modes must be considered when analysing the stability of the solution, as in [1].

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The only possible WZ interaction which can arise between the Dp'-brane and the background is a coupling to the dual, $\tilde{C}_{p'}$, whose field strength, $\tilde{F} = d\tilde{C}_{p'}$ is the Hodge dual of the field strength of the background $(p+1)$-form, $F = dC_p$. For this coupling to be possible $\tilde{C}_{p'}$ must be a $(p'+1)$-form, and so requires $p + p' = 6$. For an arbitrary external p'-form $\tilde{C}_{p'}$, after gauge fixing and assuming rigid motion the WZ action becomes

$$S_{WZ} = -mq \int dt \left(\tilde{C}_{0\cdots p'} + \tilde{C}_{a1\cdots p'}\,\dot{X}^a\right) \qquad a = p'+1, \ldots, 9\,, \tag{2.6}$$

where $q = 1$ for a brane and $q = -1$ for an antibrane. Furthermore, assuming $\tilde{C}_{p'} = \tilde{C}_{p'}(z)$ for some subset, z, of the coordinates, the curvature must have nonzero components $\tilde{F}_{01\ldots p'z}$ and $\tilde{F}_{a1\ldots p'z}$. Being the Hodge dual of the $F_{0\ldots p}$ curvature sourced by the background brane, it cannot have components in the 0 direction, and so $\tilde{F}_{01\ldots p'z} = 0$, which in turn implies that $\tilde{C}_{0\ldots p'}$ must vanish. Moreover, since the $1, \ldots, p'$ directions of the probe are parallel to some of the $1, \ldots, p$ directions of the background, the dual field strength, $\tilde{F}_{a1\ldots p'z}$, cannot have components in these directions. From this last statement we deduce that $\tilde{C}_{a1\ldots p'} = 0$ if $p' \neq 0$. We conclude that a nonvanishing WZ term is possible only when $p' = 0$ and then only if $p = 6$.

2.2 Equations of motion

We next examine the equations of motion for the probe brane, and with the previous paragraph in mind we divide the discussion into two cases, depending on whether or not there is a WZ contribution to the probe-brane action. When discussing the various possibilities it is worth keeping in mind that our choice of background fields assumes $p < 7$ and that D-branes are all odd- or all even-dimensional within any particular string theory, and so we may take $p - p'$ to be even. It follows that the difference $p - p'$ can take the possible values $0, 2, 4$ and 6, although we exclude the case $p - p' = 0$ here, because this is the case studied in the branonium analysis [1].

Born infeld only. We first assume $p' \neq 0$ or $p \neq 6$, so that the WZ term does not appear in the probe-brane action. Since this excludes the case $p - p' = 6$, we need consider only $p - p' = 2$ or 4 (with this last possibility corresponding to the supersymmetric case if the branes are at rest).

Rotational invariance in the y_n space ensures the conservation of angular momentum, $L_{mn} = y_m p_n - y_n p_m$, and so we can describe the plane in which the motion takes place using polar coordinates, r, φ. The action (2.5) then becomes

$$S_{BI} = -m \int dt\, h^{(p-p'-4)/4} \sqrt{1 - \dot{x}_i^2 - h(\dot{r}^2 + r^2\dot{\varphi}^2)} \qquad i = p'+1, \ldots, p\,, \tag{2.7}$$

and the canonical momenta are

$$p_i = m\,\frac{h^{(p-p'-4)/4}}{\sqrt{(\cdots)}}\dot{x}_i \equiv m\,u_i$$
$$p_r = m\frac{h^{(p-p')/4}}{\sqrt{(\cdots)}}\dot{r} \equiv m\,\rho$$

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$$p_\varphi = m\frac{h^{(p-p')/4}}{\sqrt{(\cdots)}}r^2\dot{\varphi} \equiv m\,\ell\,, \tag{2.8}$$

where $\sqrt{(\cdots)}$ stands for the square root in (2.7), and the right-hand-most equalities define the variables u_i, ρ and ℓ.

Given these canonical momenta we can build the conserved hamiltonian, which turns out to be

$$E = m\sqrt{h^{(p-p'-4)/2} + u_i^2 + \frac{1}{h}\left(\rho^2 + \frac{\ell^2}{r^2}\right)} \equiv m\varepsilon\,. \tag{2.9}$$

Since this is a monotonically increasing function of the canonical momenta, it is bounded from below by the following 'potential':

$$V \equiv \varepsilon(\rho = u_i = \ell = 0) = h^{(p-p'-4)/4} \tag{2.10}$$

which, as expected, is a constant only in the supersymmetric case $p - p' = 4$.[3] At long distances, $r \gg l_s$, the potential goes as $V \sim 1 + \frac{1}{4}(p - p' - 4)k/r^{7-p}$. Notice that this potential is attractive (and so may have bound states) if $p - p' < 4$ and repulsive if $p - p' > 4$ (as will be shown in the next section), conditions which we have seen also define when an open-string tachyon exists.

Translation invariance within the dimensions parallel to the Dp-brane and rotational invariance in the dimensions transverse to it ensure that the momenta u_i and ℓ are conserved. Their conservation allows the construction of an effective potential for the radial motion in the usual way, with the result

$$V_{\text{eff}} \equiv \varepsilon(\rho = 0) = \sqrt{h^{(p-p'-4)/2} + u_i^2 + \frac{\ell^2}{hr^2}}\,. \tag{2.11}$$

See figures 2, 3 and 4 for plots of the shape of this potential given various choices for p and p'. This potential shows qualitatively when bound orbits can occur, since they do if the effective potential ever passes below its asymptotic value at $r \to \infty$. Circular orbits occur at the special radii which are minima of V_{eff}.

To determine when closed orbits exist we therefore explore the shape of the potential near the origin and at infinity. In the limit[4] $r \to 0$ we can replace h by k/r^{7-p}, obtaining

$$V_{\text{eff}} = \sqrt{b\,r^{(p-7)(p-p'-4)/2} + u_i^2 + \frac{\ell^2 r^{5-p}}{k}}\,, \qquad b = k^{(p-p'-4)/2}\,, \tag{2.12}$$

and since we assume $p < 7$, the sign of the exponent in the first term is given by the sign of $4 + p' - p$ (i.e. it is positive when the interaction is attractive and negative when it is repulsive). We find in this way the following limits:

[3]In order to see the vanishing of V in the case $p = p'$ one must also include the Wess-Zumino term, which gives an extra contribution to the potential of the form $-qh^{-1}$, where $q = 1$ for a brane and $q = -1$ for the antibrane.

[4]Of course, if r is taken all of the way to l_s these expressions all receive string corrections, which become important. Our interest is in r not quite this small, to sketch the shape of the potential.

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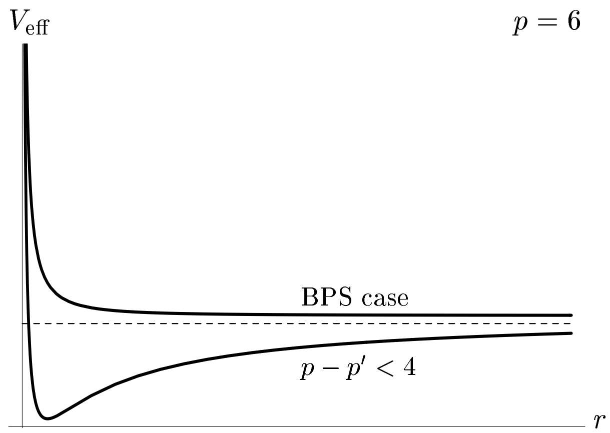

Figure 2: Effective potential for the case $p = 6$, in the BPS ($p' = 2$) and non-BPS ($p' = 4$) cases.

- For $p = 6$ the last term ensures that the potential diverges at the origin.
- For $p = 5$ the last term tends to a constant for small r. For $p' = 3$ the potential converges to $\sqrt{u_i^2 + \ell^2/m}$ and if $p' = 1$ (the BPS case) it instead approaches $\sqrt{1 + u_i^2 + \ell^2/m}$.
- For $p < 5$, the last term vanishes and at the origin the potential converges to $\sqrt{u_i^2}$ if $p - p' = 2$, or to $\sqrt{1 + u_i^2}$ in the supersymmetric case $p - p' = 4$ (i.e. $p = 4$, $p' = 0$).

On the other hand, in the large-r limit, we instead find

$$V_{\rm eff} = \sqrt{1 + u_i^2 + \frac{(p - p' - 4)k}{2\, r^{7-p}} + \frac{\ell^2}{r^2}}\,, \tag{2.13}$$

and we see that the potential asymptotes to the constant $\sqrt{1 + u_i^2}$ when $r \to \infty$. This limit corresponds to the expected dispersion relation for motion parallel with the Dp-brane, for which $E = \sqrt{m^2 + p_i^2}$ and so $\varepsilon = \sqrt{1 + u_i^2}$. How this limit is approached depends on p and p', in the following ways.

- In the supersymmetric case, $p - p' = 4$ the limit is reached from above, regardless of the value of p.
- If $p = 6$, the non-supersymmetric case is $p' = 4$ and so the third term dominates. This implies the potential approaches its limit from below.
- For $p = 5$ it is the choice $p' = 3$ which is not supersymmetric. In this case the limiting value is reached from below or from above depending on the sign of $\ell^2 + (p - p' - 4)k/2 = \ell^2 - k$.
- If $p < 5$, the last term always dominates and the potential approaches its limit from above.

Combining the behaviour for large and small r the following picture emerges.

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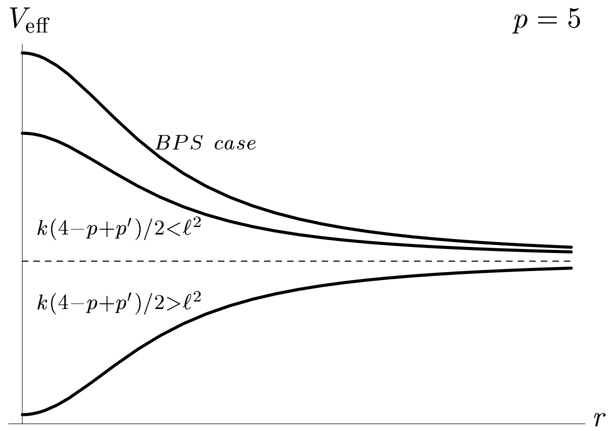

Figure 3: Effective potential for the $p = 5$ case, depending on the value of the angular momentum.

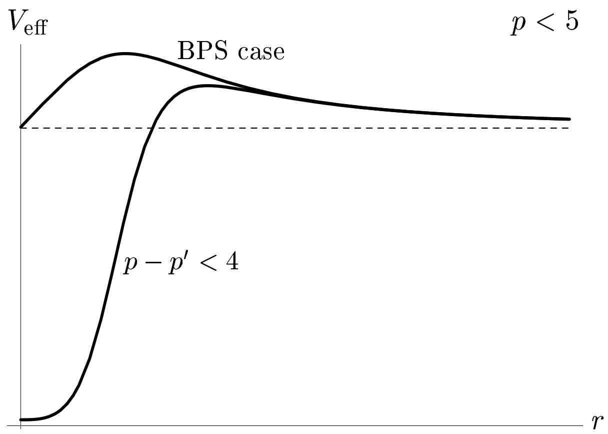

Figure 4: Form of the effective potential when $p < 5$.

- If $p = 6$ and $p' = 4$ the V_{eff} reaches a minimum away from $r = 0$, and so bound orbits exist which are bounded away from $r = 0$. In the supersymmetric case where $p = 6$ and $p' = 2$ no such minimum or bound states arise.

- If $p = 5$ and $p' = 1$ (the BPS case) there is no bound state. If $p' = 3$ the existence of a minimum depends on the angular momentum, and there is a bound orbit if $\ell^2 < (4 - p + p')k/2 = k$.

- If $p < 5$ the potential has a minimum at the origin. For the supersymmetric case ($p = 4$, $p' = 0$) V has the same value at this minimum as it does at infinity. Otherwise V is smaller at $r = 0$ than at infinity. In either case classically localized orbits exist.

In terms of the conserved quantities u_i, ℓ and ε, the equations of motion reduce to first-order conditions

$$\dot{x}_i = \frac{u_i}{\varepsilon}$$

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p	p'	A	B	C
5	1	$1+k(1+u_i^2-\varepsilon^2)/\ell^2$	0	$(\varepsilon^2-u_i^2-1)/\ell^2$
5	3	$1+k(u_i^2-\varepsilon^2)/\ell^2$	0	$(\varepsilon^2-u_i^2-1)/\ell^2$
6	2	1	$k(\varepsilon^2-u_i^2-1)/\ell^2$	$(\varepsilon^2-u_i^2-1)/\ell^2$
6	4	1	$k(\varepsilon^2-u_i^2)/\ell^2$	$(\varepsilon^2-u_i^2-1)/\ell^2$

Table 1: The constants appearing in the orbit integral when trajectories are conic sections.

$$\begin{aligned}\dot r &= \frac{\rho}{\varepsilon h} = \frac{1}{\varepsilon h}\sqrt{(\varepsilon^2-u_i^2)h - h^{(p-p'-2)/2} - \frac{\ell^2}{r^2}} \\ \dot\varphi &= \frac{\ell}{\varepsilon h r^2}\,. \end{aligned} \tag{2.14}$$

These equations of motion are easily integrated to obtain the orbits

$$\varphi - \varphi_0 = \int \frac{\ell\, dx}{\sqrt{(\varepsilon^2-u_i^2)h - h^{(p-p'-2)/2} - \ell^2 x^2}}\,, \tag{2.15}$$

where the integration variable is $x = 1/r$. Since $p-p' = 2$ or 4, the power of h appearing here is $(p-p'-2)/2 = 0$ or 1. Consequently, for $p < 6$ the quantity inside the square root is a polynomial of degree $7-p$ in the variable x. It is quadratic for both $p = 5$ and $p = 6$.

When the argument of the square root is quadratic in x the integral of (2.15) reduces to

$$\varphi - \varphi_0 = \int \frac{dx}{\sqrt{-Ax^2+Bx+C}} \tag{2.16}$$

for appropriate constants A, B and C (which are listed in table 1), then the integral may be performed explicitly to give the standard conic sections of the Kepler problem. For instance, if $B \neq 0$ (which from the table requires $p = 6$) the bound orbits are ellipses,

$$r = \frac{a(1-e^2)}{1+e\cos(\sqrt{A}\varphi)}\,, \tag{2.17}$$

where we define $a = -B/(2C)$ and $e^2 = 1 + 4AC/B^2$. The condition for a bound orbit is $e < 1$, i.e. $C < 0$, and this can be seen from the table to require energies below the asymptotic value of the potential V_{eff}: $\varepsilon < \sqrt{1+u_i^2}$. Since $V_{\text{eff}} > 1+u_i^2$ for all r when $p' = 2$, we see that Keplerian bound orbits arise only in the case $p = 6$ and $p' = 4$.

For the case $p = 5$, the orbits are also given by the integral (2.16), but with $B = 0$. If $A < 0$ and $C < 0$, we instead have bound (non closed) orbits only if $p' = 3$ and $\ell^2 < k$, with the form

$$r = \sqrt{\frac{A}{C}}\frac{1}{\cosh(\sqrt{-A}\varphi)}\,. \tag{2.18}$$

The parameter C can be negative only in the non-BPS case $p' = 3$. The sign of A depends on ℓ and will be negative when $\ell^2 < (\varepsilon^2 - u_i^2)k$, which is always the case if the potential is attractive $\ell^2 < k$ and the energy is less than the value of the potential at infinity $\varepsilon < \sqrt{1+u_i^2}$.

The integral, eq. (2.15), is similarly integrable in terms of elliptic functions in the cases $p = 1, 3, 4$.

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The Wess-Zumino term. We now return to the special case for which a probe-brane coupling with a WZ term is possible: $p = 6$ and $p' = 0$. In this case the curvature sourced by the background brane has components $F_{0\ldots 6r} = -k/(r^2h^2)$, and so the dual is $\tilde{F}_{\theta\varphi} = -k\sin\theta$, where r, θ and φ are spherical polar coordinates in the three dimensions transverse to the D6-brane. The gauge potential which produces this dual field strength then is $\tilde{A}_\varphi = k(\cos\theta - 1)$, which corresponds to that of a magnetic monopole and we have fixed the constant of integration by demanding that the Dirac string is along the negative z axis. This leads to the WZ term

$$S_{WZ} = -qkm\int dt\,(\cos\theta - 1)\,\dot{\varphi}\,. \tag{2.19}$$

This action arises more prosaically in the low-energy description of spin waves in ferromagnets [4]. The complete action then reads

$$S = -m\int dt\left[h^{1/2}\sqrt{1 - \dot{x}_i^2 - h(\dot{r}^2 + r^2(\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2))} + qk(\cos\theta - 1)\dot{\varphi}\right]. \tag{2.20}$$

In the absence of the WZ term the D0-brane motion would be confined to a plane by conservation of angular momentum, $\vec{L} \propto \vec{y}\times\dot{\vec{y}}$, transverse to the D6-brane. Choosing the z axis to be in the $\vec{L}$ direction then would allow the D0-brane motion to be restricted to the equatorial plane, $\theta = \pi/2$. The same is not true once the Wess-Zumino term is included, because this term is not separately invariant under 3-dimensional rotations and gauge transformations, but is invariant under a combination of both. Therefore, $\vec{L}$ is no longer conserved and the motion need no longer be restricted to lie on a plane. The conserved quantity in this case is a generalized angular momentum, given by

$$\vec{J} = \frac{h^{3/2}}{\sqrt{(\cdots)}}\,\vec{y}\times\dot{\vec{y}} + qk\check{y}\,, \tag{2.21}$$

where $\check{y} = \vec{y}/|\vec{y}|$ is the unit vector in the $\vec{y}$ direction. Choosing $\vec{J}$ to define the z axis, the implications of $\vec{J}$ conservation are most easily seen by using the observation that $\vec{J}\cdot\check{y} = J\cos\theta = qk$ is a constant of motion, and so the trajectories are restricted to be at a fixed value of θ, given by $\cos\theta = qk/J$. That is, $\vec{y}$ precesses around a cone of opening angle θ whose axis is in the $\vec{J}$ direction. Notice that this cone becomes the plane $\theta = \pi/2$, as expected, if we put $q = 0$.

For motion on this cone the action becomes

$$S = -m\int dt\left[h^{1/2}\sqrt{1 - \dot{x}_i^2 - h(\dot{r}^2 + r^2\sin^2\theta\dot{\varphi}^2)} + qk(\cos\theta - 1)\dot{\varphi}\right]. \tag{2.22}$$

where $\cos\theta$ and $\sin\theta$ should be regarded as constants which are determined from the relation $\cos\theta = qk/J$. The canonical momenta are easily derived

$$\begin{aligned} u_i &= \frac{h^{1/2}}{\sqrt{(\cdots)}}\dot{x}_i \\ \rho &= \frac{h^{3/2}}{\sqrt{(\cdots)}}\dot{r} \\ \ell &= \frac{h^{3/2}}{\sqrt{(\cdots)}}r^2\sin^2\theta\dot{\varphi} - qk(\cos\theta - 1)\,, \end{aligned} \tag{2.23}$$

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and, as before, u_i and ℓ are conserved because the action is independent of x^i and φ. The conserved hamiltonian is

$$\varepsilon = \sqrt{h + u_i^2 + \frac{1}{h}\left(\rho^2 + \frac{1}{r^2}\left(\frac{\ell - qk(\cos\theta - 1)}{\sin\theta}\right)^2\right)}\,, \tag{2.24}$$

and the potential derived from this hamiltonian is given by

$$V \equiv \varepsilon(\rho = u_i = \ell = 0) = \sqrt{h + \frac{1}{hr^2}\left(\frac{k(\cos\theta - 1)}{\sin\theta}\right)^2} = \sqrt{h + \frac{k^2}{hr^2}\left(\frac{J - qk}{J + qk}\right)}\,, \tag{2.25}$$

which is repulsive, as stated in the previous section.

The equations of motion obtained from the above hamiltonian are

$$\begin{aligned} \dot{x}_i &= \frac{u_i}{\varepsilon} \\ \dot{r} &= \frac{\rho}{h\varepsilon} = \frac{1}{h\varepsilon}\sqrt{(\varepsilon^2 - u_i^2)h - h^2 - \frac{\hat{\ell}^2}{r^2}} \\ \dot{\varphi} &= \frac{1}{h\varepsilon}\,\frac{\hat{\ell}}{\sin\theta\, r^2} \end{aligned} \tag{2.26}$$

with $\hat{\ell} = [\ell - qk(\cos\theta - 1)]/\sin\theta$. Integrating these the following orbits are obtained

$$\varphi - \varphi_0 = \int \frac{dx}{\sqrt{-\mathcal{A}x^2 + \mathcal{B}x + \mathcal{C}}}\,. \tag{2.27}$$

with $\mathcal{A} = (1 + k^2/\hat{\ell}^2)\sin^2\theta$, $\mathcal{B} = (\varepsilon^2 - u_i^2 - 2)k\sin^2\theta/\hat{\ell}^2$ and $\mathcal{C} = (\varepsilon^2 - u_i^2 - 1)\sin^2\theta/\hat{\ell}^2$.

We see that the motion is in the surface of a cone whose axis is given by $\vec{J}$, with the source brane at the origin. The trajectories can be obtained directly from eq. (2.27), which, since the argument of the square root is quadratic in x, can be easily integrated to give conic sections (i.e. parabolic or hyperbolic motion) on the plane perpendicular to $\vec{J}$.

2.3 Equivalent non-relativistic lagrangians

Since the orbits of these relativistic systems can be explicitly expressed in terms of elementary integrals, it is possible to identify a non-relativistic lagrangian of the form $\mathcal{L}_{\rm eq} = \frac{1}{2}\dot{\vec{y}}^2 - V_{\rm eq}(\vec{y})$ whose classical trajectories precisely coincide with the trajectories of the fully relativistic system. We call the lagrangian of this equivalent system, the *equivalent non-relativistic lagrangian.* The existence of this system is useful, since general results often exist for these non-relativistic systems which are derived under the assumption of trivial kinetic energies. Although these cannot be directly applied to the relativistic D-brane systems, due to the non-trivial velocity-dependent interactions which they contain, their implications *can* be applied to the equivalent non-relativistic system, and so carried over to the trajectories of the fully relativistic system.

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Born-Infeld only. In the Born-Infeld case, the (fully relativistic) trajectories are given by eq. (2.15). These same trajectories also give the orbits for the system defined by the non-relativistic lagrangian having the following form

$$\mathcal{L}_{\rm eq}(r) = \frac{1}{2}(\dot{r}^2 + r^2\dot{\varphi}^2) + \frac{k_{p-p'}}{r^{7-p}}\,, \tag{2.28}$$

for which $V_{\rm eq} = -k_{p-p'}/r^{7-p}$. The orbits of this system agree with the relativistic one provided we make the identifications $k_2 = (\varepsilon^2 - u_i^2)k/2$, $k_4 = (\varepsilon^2 - u_i^2 - 1)k/2$ and identify the conserved energy as $\varepsilon_{\rm eq} = (\varepsilon^2 - u_i^2 - 1)/2$.

As mentioned above, the utility of identifying this equivalent system comes from the general results which are available in the non-relativistic case. For instance [5], general central force problems, $V_{\rm eq} \propto r^\xi$ are known to have solutions in terms of trigonometric functions when the exponent is $\xi = 2, -1$ or -2. This corresponds in the brane case to $p = 5$ or 6 since $\xi = p - 7$. Alternatively, the solution may be written in terms of elliptic functions when $\xi = 6, 4, 1, -3, -4$ or -6 (corresponding to $p = 1, 3$ and 4). Furthermore, the general result known as Bertrand's theorem states that the only potentials giving rise to closed orbits are the Kepler problem, $\xi = -1$ (or $p = 6$), and the harmonic oscillator, $\xi = 2$ (which would be $p = 8$, and so has no analogue within the domain of our approximations). This allows us to conclude that only for $p = 6$ can branes have closed orbits, and then only when the energy is negative, $\varepsilon_{eq} < 0$. Looking at eq. (2.9), we see that, as expected, this last condition can be satisfied only if $p' = 4$ and is never realized in the BPS case $p' = 2$. Since the orbits are non precessing, we know that we must have some conserved vector analogous to the Runge-Lenz vector of the nonrelativistic system, this vector can be found in a straightforward manner following similar steps as for the branonium case [1].

Notice that the equivalent non-relativistic lagrangian has a nontrivial potential even for the BPS case where $p - p' = 4$, despite the vanishing of the potential in the relativistic problem. The existence of the non-relativistic potential reflects the fact that the trajectories of the non-relativistic system must agree with the exact trajectories of the relativistic system, and so the non-relativistic potential must encode the velocity-dependent interactions of the moving D-brane system. Notice that k_4 (and so also $V_{\rm eq}(r)$) does vanish for Dp'-branes which move only parallel to the Dp-brane, since in this case $\varepsilon^2 = 1 + u_i^2$. In particular it also vanishes for completely static branes, for which $u_i = 0$.

These observations allow a simple evaluation of the trajectories of BPS branes moving only under the influence of velocity-dependent forces. For example, as applied to the BPS brane-brane system with $p = p'$ of ref. [1], the trajectories of the fully-relativistic system are given by elementary integrals, which reduce to conic sections (hyperbolae and parabolae) when $p = 6$.

Remarkably, in the non BPS cases $p - p' = 2$ the potential of the equivalent non-relativistic lagrangian has the same functional form as the potential which arises in the non-relativistic limit of the full lagrangian. As a consequence of this, the functional form of the trajectories do not change as one crosses over to the relativistic from the non-relativistic regimes. Instead, the only change is how orbital parameters (such as eccentricity and semi-major axis) depend on physical quantities (like energy and angular momentum).

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Defining the non-relativistic limit by $\varepsilon \approx 1 + \varepsilon_{NR}$, with $\varepsilon_{NR} \sim u_i^2 \sim \ell^2/r^2 \sim k/r^{7-p} \ll 1$, we see that in this limit $k_2 \approx k/2$, and so $V_{\rm eq}(r)$ smoothly goes over to $V_{NR}(r)$. This is as might have been expected, given that the equivalent non-relativistic system must incorporate in particular the non-relativistic trajectories of the full D-brane problem. A similar correspondence was also found for brane-antibrane motion with $p = p'$ [1].

The same equivalence of functional form of $V_{\rm eq}(r)$ and the non-relativistic potential, $V_{NR}(r)$, *cannot* hold for the BPS case, since we've just seen that $V_{\rm eq}(r)$ is nonzero, while we know the non-relativistic limit of the BPS problem has $V_{NR}(r) \equiv 0$. Instead in this case we only have $V_{\rm eq}(r) \to V_{NR}(r) = 0$ in the non-relativistic limit, since $k_4 \approx 0$.

The Wess-Zumino term. The (fully relativistic) orbits for the D6-D0 system including the Wess-Zumino term are also the same as those obtained from an equivalent non-relativistic problem, which in this case corresponds to the lagrangian of an electric charge in a field generated by a dyon. The equivalent lagrangian is given by

$$\mathcal{L}_{\rm eq} = \frac{1}{2}(\dot{r}^2 + r^2 \sin^2\theta_{\rm eq}\dot{\varphi}^2) + \frac{k_6}{r} - q\, k_6(\cos\theta_{\rm eq} - 1)\dot{\varphi} \tag{2.29}$$

This problem has been studied in detail in the past, see for instance [6]. The orbits obtained for this equivalent system are given in terms of the conserved quantities $\ell_{\rm eq}$ and $\varepsilon_{\rm eq}$ by

$$\varphi - \varphi_0 = \int \frac{dx}{\sqrt{-\mathcal{A}_{\rm eq}x^2 + \mathcal{B}_{\rm eq}x + \mathcal{C}_{\rm eq}}}\,, \tag{2.30}$$

where $\mathcal{A}_{\rm eq} = \sin^2\theta_{\rm eq}$, $\mathcal{B}_{\rm eq} = 2k_6 \sin^2\theta_{\rm eq}/\hat{\ell}^2_{\rm eq}$, $\mathcal{C}_{\rm eq} = 2\varepsilon_{\rm eq}\sin^2\theta_{\rm eq}/\hat{\ell}^2_{\rm eq}$ and $\hat{\ell}_{\rm eq} = [\ell_{\rm eq} - qk_6(\cos\theta_{\rm eq} - 1)]/\sin\theta_{\rm eq}$. These integrals coincide with the fully relativistic ones, (2.27), if we identify $k_6 = (\varepsilon^2 - u_i^2 - 2)k/2$, $\varepsilon_{\rm eq} = (\varepsilon^2 - u_i^2 - 1)/2$, $\hat{\ell}^2_{\rm eq} = \hat{\ell}^2 + k^2$ and $\sin^2\theta_{\rm eq} = \sin^2\theta(1 + k^2/\hat{\ell}^2)$. Nevertheless, it should be noted that this is not a complete identification of the relativistic and the non-relativistic orbits, but only of their projections into the (x, y) plane, because we are changing the angle of the cone in which the motion takes place $\theta \neq \theta_{eq}$.

2.4 Quantum dynamics

Let us study the same problem from the quantum mechanical point of view. In the absense of a WZ term, the hamiltonian will be given by (2.9) and then the quantum mechanical wave function satisfy

$$\sqrt{h^{(p-p'-4)/2} + u_i^2 + \frac{1}{h}\left(\rho^2 + \frac{\ell^2}{r^2}\right)}\,\Psi = \varepsilon\Psi\,. \tag{2.31}$$

Squaring the hamiltonian operator we find

$$\left(h^{(p-p'-4)/2} + u_i^2 + \frac{1}{h}\left(\rho^2 + \frac{\ell^2}{r^2}\right)\right)\Psi = \varepsilon^2\Psi\,. \tag{2.32}$$

Taking into account that $p - p'$ can only be 2 or 4, we can rewrite the above equation in the form

$$\left(\frac{1}{2}\left(\rho^2 + \frac{\ell^2}{r^2}\right) - \frac{k_{p-p'}}{r^{7-p}}\right)\Psi = \varepsilon_{eq}\Psi\,. \tag{2.33}$$

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where ε_{eq} and $k_{p-p'}$ are defined as in the previous section. Then, as in the $p = p'$ case, we recover the equivalence of our fully relativistic problem with a non-relativistic one at the quantum level.

A similar treatment can be done for the $p = 6,\ p' = 0$ case, in this case the Schrödinger equation turns out to be:

$$\left(\frac{1}{2}\left(\rho^2 + \frac{\hat{\ell}^2 + k^2}{r^2}\right) - \frac{k(\varepsilon^2 - \mu_i^2 - 2)}{2r}\right)\Psi = \frac{\varepsilon^2 - \mu_i^2 - 1}{2}\Psi\,. \tag{2.34}$$

The equivalence with the corresponding non-relativistic system can be read by making the same identifications as in the classical case in the previous section. Notice that now, even though there is a mapping to the corresponding equation of the non-relativistic system, the mapping includes a shift in the angular momentum $\hat{\ell}$. This is what prevents the complete identification of the classical orbits, and it does not occur in the $p-p' = 2, 4$ cases, the main technical reason for it is that in this case there is a h^2 term is the square root in (2.27) that does not appear in the other cases. A similar situation occurs in hydrogenoid atoms [8].

Notice that in all cases, this naive quantum treatment is limited if, as we expect, the D-branes involved have macroscopic sizes. The corresponding Bohr radius is inversely proportional to the mass of the probe branes and therefore this naturally sets us to the classical regime. The only exceptions would be if the branes extend only over compactified dimensions that could be either the compact dimensions of compactified string theory, or the spacetime dimensions in the very early universe. It would be interesting to explore if in those cases this quantum behaviour has any physical implications.

3. Cheshire branes: uplifting to 11 dimensions

It is well known that type-IIA strings may be obtained by compactifying M-Theory on a circle, but there are no branes in M theory to which D6-branes can lift. They do not lift to 11 dimensions as branes at all, but instead as particular multi Taub-NUT spaces. We break from the previous line of development in this section to use this lift to provide a complementary analysis of branonium-like systems in M-theory.

3.1 The taub-NUT background and probes

The first step is to define the 11-dimensional field configuration which corresponds to the uplift of a stack of source D6-branes (together with the fields to which they give rise).

The background. The eleven dimensional background obtained when uplifting N D6-branes is purely gravitational, since the ten-dimensional dilaton, Ramond-Ramond 1-form and the metric are encoded in the 11 dimensional metric as follows:

$$ds^2_{(11)} = ds^2_{Mink7} + h\, d\vec{x}\cdot d\vec{x} + h^{-1}(d\psi + \vec{A}\cdot d\vec{x})^2\,. \tag{3.1}$$

Here ds^2_{Mink7} represents seven-dimensional flat space, and the remaining part of the metric is a four-dimensional euclidean Taub-NUT metric. The ψ coordinate is the 11th dimension,

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and is a periodic variable with period $4\pi l_s$, while h and $\vec{A}$ are given by

$$h = 1 + \frac{k}{r}, \qquad \vec{\nabla} \times \vec{A} = \vec{\nabla} h\,. \tag{3.2}$$

where $k = g_s l_s N/2$. The expression for $\vec{A}$ can be made more explicit by writing instead $A_\varphi = k\,\cos\theta$.

Probe brane actions. The probes available in M-theory are M2-branes and M5-branes which from the IIA point of view can describe D2-branes, F1-strings, NS5-branes or D4-branes, depending on whether or not these M-branes are transverse to or wrap the M-theory dimension.

Let us consider a general Nambu-Goto type of action for the M-theory branes in a purely geometric background:

$$\begin{aligned} S \;=\; -M_{p'} \int d^{p'+1}\xi \sqrt{|g_{AB}(X)\,\partial_\alpha X^A\,\partial_\beta X^B|} \\ \alpha, \beta = 0, \ldots, p'\,; \qquad A, B = 0, \ldots, 10\,. \end{aligned} \tag{3.3}$$

As in the previous section we can gauge-fix the reparameterization invariance using static gauge and assume rigid motion, $X^a = X^a(t)$, to get

$$\begin{aligned} S \;=\; -m \int dt\, \sqrt{|g_{\alpha\beta}(X) + g_{a(\alpha}(X)\,\delta^0_{\beta)}\dot{X}^a + g_{ab}(X)\dot{X}^a\dot{X}^b\,\delta^0_\beta\delta^0_\alpha|} \\ \alpha, \beta = 0, \ldots, p'\,; \qquad a, b = p'+1, \ldots, 10\,. \end{aligned} \tag{3.4}$$

There are two cases to consider, depending on whether the branes are positioned at a single point in the ψ dimension, or whether they instead wrap this dimension.

3.2 D2 and NS5 branes

We now specialize to an M-theory brane which is located at a point in the 11th dimension, and so can describe a D2-brane or an NS5-brane as seen from the type-IIA point of view.

The action. In this section we focus on the case of a $(p'+1)$-dimensional probe with $p' \leq 6$, and we assume that the $p'+1$ directions along which it extends are transverse to the Taub-NUT space. The choices $p' = 2$ and $p' = 5$ then describe the two choices for M-theory branes. We have then $g_{\alpha\beta}(X) = \eta_{\alpha\beta}$ and the probe-brane action becomes

$$\begin{aligned} S = -m \int dt\, \sqrt{1 - \dot{x}_i^2 - h(\dot{r}^2 + r^2(\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2)) - h^{-1}(\dot{\psi} + k\cos\theta\,\dot{\varphi})^2} \\ i = p'+1, \ldots, 6\,. \end{aligned} \tag{3.5}$$

Using rotational invariance to place the plane of motion at $\theta = \pi/2$, we have

$$S = -m \int dt\, \sqrt{1 - \dot{x}_i^2 - h(\dot{r}^2 + r^2\dot{\varphi}^2) - h^{-1}\dot{\psi}^2}\,. \tag{3.6}$$

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Equations of motion. The canonical momenta derived from the action just given are

$$\begin{aligned}
p_i &= \frac{m}{\sqrt{(\cdots)}}\,\dot{x}_i \equiv, m\,u_i \\
p_r &= \frac{m}{\sqrt{(\cdots)}}\,h\dot{r} \equiv, m\,\rho \\
p_\varphi &= \frac{m}{\sqrt{(\cdots)}}\,hr^2\dot{\varphi} \equiv, m\,\ell \\
p_\psi &= \frac{m}{\sqrt{(\cdots)}}\,h^{-1}\dot{\psi} \equiv, m\,\tilde{\ell}\,,
\end{aligned} \tag{3.7}$$

where now $\sqrt{(\cdots)}$ stands for the square root in (3.6). In terms of these the energy becomes

$$E = m\sqrt{1 + u_i^2 + \left(\rho^2 + \frac{1}{h}\frac{\ell^2}{r^2}\right) + \tilde{\ell}^2 h} \equiv m\,\varepsilon\,. \tag{3.8}$$

The equations of motion for this system then reduce to the following first-order equations:

$$\begin{aligned}
\dot{x}_i &= \frac{u_i}{\varepsilon} \\
\dot{r} &= \frac{\rho}{\varepsilon h} = \frac{1}{\varepsilon h}\sqrt{(\varepsilon^2 - u_i^2 - 1)h - \frac{\ell^2}{r^2} - \tilde{\ell}^2 h^2} \\
\dot{\varphi} &= \frac{\ell}{\varepsilon h r^2} \\
\dot{\psi} &= \frac{\tilde{\ell} h}{\varepsilon}\,.
\end{aligned} \tag{3.9}$$

These integrate to give the following trajectories:

$$\begin{aligned}
\varphi - \varphi_0 &= \int \frac{\ell dx}{\sqrt{(\varepsilon^2 - u_i^2 - 1)h - \tilde{\ell}^2 h^2 - \ell^2 x^2}} \\
\psi - \psi_0 &= \int \frac{h^2 \tilde{\ell} dx}{x^2\sqrt{(\varepsilon^2 - u_i^2 - 1)h - \tilde{\ell}^2 h^2 - \ell^2 x^2}}\,.
\end{aligned} \tag{3.10}$$

Since h is linear in $x = 1/r$, it is clear that the first of these integrals gives conical sections. It is also easy to check that there are no bound states, and so all orbits are parabolae or hyperbolae as expected from the analysis of D6-D2 system in type IIA. Nor can Kaluza-Klein momenta change this conclusion.

Ten dimensional point of view. How does the above picture compare with the same problem as studied from a ten-dimensional point of view? As we have previously seen, after gauge fixing and assuming rigid motion the action for a Dp'-brane moving in a D6-brane background (with $p' \neq 0$) is

$$S = -m\int dt\, h^{(2-p')/4}\sqrt{1 - \dot{x}_i^2 - h(\dot{r}^2 + r^2\dot{\varphi}^2)}\,, \tag{3.11}$$

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which *differs* from the action (3.6) obtained from the eleven dimensional point of view by the presence of the factor $h^{(2-p')/4}$ outside the square root, and by the absence of the last term, $h^{-1}\dot{\psi}^2$, inside the square root.

D2-branes: the first of these differences is absent if we specialize to the D2-brane case, for which $p' = 2$, as we now do. To eliminate the second difference, we turn on an electromagnetic field on the brane and rewrite the ten-dimensional D2-brane action as follows [17]:

$$S = -T_2 \int d^3\xi \left(e^{-\phi}\sqrt{|g_{\alpha\beta} + e^{2\phi} t_\alpha t_\beta|} + \frac{1}{2}\epsilon^{\alpha\beta\gamma} t_\alpha F_{\beta\gamma} \right) \tag{3.12}$$

which is equivalent to the usual BI form after eliminating t_α using its equations of motion.

We now choose a purely magnetic field, $F_{12} = B$, and evaluate using the D6-brane background (2.1). In this case, the eqs. of motion for t_α imply $t_1 = t_2 = 0$ and we have

$$S = -m \int dt \left(\sqrt{1 - \dot{x}_i^2 - h(\dot{r}^2 + r^2\dot{\varphi}^2) - h^{-1} t_0^2} + t_0 B \right). \tag{3.13}$$

Integrating the last term by parts, the equations of motion for A_1 and A_2 imply $\partial_1 t_0 = \partial_2 t_0 = 0$, and so we define $t_0 = t_0(t) \equiv \dot{\psi}$. We obtain in this way

$$S = -m \int dt \sqrt{1 - \dot{x}_i^2 - h(\dot{r}^2 + r^2\dot{\varphi}^2) - h^{-1}\dot{\psi}^2}\,, \tag{3.14}$$

which recovers the action obtained above from the purely eleven-dimensional point of view.

The relation between the magnetic field and our eleven-dimensional variables is obtained from the equation of motion for t_0 (now called $\dot{\psi}$) derived from (3.13), which is

$$\tilde{\ell} = \frac{h^{-1}\dot{\psi}}{\sqrt{(\cdots)}} = B\,. \tag{3.15}$$

We see that the magnetic field on the D2-brane, as seen from the ten-dimensional point of view, corresponds in the eleven-dimensional picture to the momentum of the membrane in the compactified direction. This is consistent with the interpretation of the Kaluza-Klein momenta, $p \sim n/g_s l_s$, in the compactified direction of M-theory as n D0-branes. These D0-branes form a bound state with the D2-branes that appears as a magnetic flux on the D2-brane with first Chern number $c_1 = n$.

NS5-branes: we now return to the second possibility: the case of an NS5-brane moving in a D6-brane background. In this case, following [9], we take the action to be

$$\begin{aligned} S &= -M \int d^4\xi\, e^{-2\phi} \sqrt{|g_{\alpha\beta} + (\dot{X}^\mu \dot{X}_\mu - e^{2\phi}\dot{y}^2)\delta^0_\alpha \delta^0_\beta|} \\ &= -m \int dt\, \sqrt{1 - \dot{x}_i^2 - h(\dot{r}^2 + r^2\dot{\phi}^2) + h^{-1}\dot{y}^2}\,. \end{aligned} \tag{3.16}$$

Here y is the world-volume scalar field as defined in [9], and we see that the identification $y = \psi - \psi_0$ leads to our original eleven-dimensional action, now corresponding to an M5-brane in a Taub-NUT background.

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3.3 F1-strings and D4-branes

The other possibility to consider is that the branes wrap the 11th dimension to produce an F1-string or a D4-brane in type-IIA string theory. We study these possibilities in this section.

The action. Taking the probe brane to be wrapped in the compact Taub-NUT direction, ψ, and evaluating the determinant in (3.4) we obtain

$$S = -m\int dt\,\sqrt{h^{-1}(1-\dot{x}_i^2)-\dot{r}^2-r^2\dot{\varphi}^2} \qquad i = p'+1,\ldots,6\,. \tag{3.17}$$

Equations of motion. Following similar steps as before, we obtain the canonical momenta

$$\begin{aligned} u_i &= \frac{h^{-1}\dot{x}_i}{\sqrt{(\cdots)}} \\ \rho &= \frac{\dot{r}}{\sqrt{(\cdots)}} \\ \ell &= \frac{r^2\dot{\varphi}}{\sqrt{(\cdots)}}\,, \end{aligned} \tag{3.18}$$

and hamiltonian

$$\varepsilon = \sqrt{u_i^2 + \frac{1}{h}\left(1+\rho^2+\frac{\ell^2}{r^2}\right)}\,. \tag{3.19}$$

These lead to the first-order equations of motion

$$\begin{aligned} \dot{x}_i &= \frac{u_i}{\varepsilon} \\ \dot{r} &= \frac{1}{\varepsilon h}\sqrt{(\varepsilon^2-u_i^2)h-1-\frac{\ell^2}{r^2}} \\ \dot{\varphi} &= \frac{\ell}{\varepsilon h r^2}\,, \end{aligned} \tag{3.20}$$

whose integration leads to the following trajectories

$$\varphi-\varphi_0 = \int\frac{\ell\,dx}{\sqrt{(\varepsilon^2-u_i^2)h-1-\ell^2x^2}}\,. \tag{3.21}$$

Ten dimensional point of view. The comparison between 10 and 11 dimensions is in this case simpler.

D4-branes: if we set $p=4$ in equation (3.11) describing a Dp-brane probe in a D6-brane background, we immediately obtain exactly (3.17). Our previous solution then represents, from the 10 dimensional point of view, a D4-brane having vanishing world-volume gauge fields, moving in a D6-brane background.

F1-strings: on the other hand, if we write the Nambu-Goto action for a fundamental string in a brane background, we obtain

$$S = -m\int dt\,\sqrt{|g_{\alpha\beta}(X)+g_{a(\alpha}(X)\,\delta_{\beta)}{}^0\dot{X}^a+g_{ab}(X)\dot{X}^a\dot{X}^b\,\delta^0_\beta\delta^0_\alpha|}\,, \tag{3.22}$$

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where, as before, we gauge fix using the static gauge and assume rigid motion. Specialized to the D6-brane background, we also immediately recover (3.17), showing that our above solution also represents a fundamental string moving in a D6-brane background.

4. Orientifold atoms

We next consider the motion of Dp-branes in the presence of orientifold planes. Orientifold planes are generic in type-II string compactifications and can interact with Dp-branes. An important difference in this case is the potential breakdown of the description of a probe brane moving in the fixed supergravity background due to the source branes, since we typically only have a single orientifold plane as a source.

4.1 Orientifold planes

We consider a configuration of N anti Dp-branes (as counted on the covering space, i.e. $N/2$ dynamical branes) and an orientifold $O^{\pm}$-plane. Each of these objects breaks one different half of the supersymmetries of the bulk. The tension and Ramond-Ramond charges of these objects are:

- anti-Dp-branes: $T = 1$ and $Q = -1$,
- Op^{+}-plane: $T = 2^{p-4}$ and $Q = 2^{p-4}$,
- Op^{-}-plane: $T = -2^{p-4}$ and $Q = -2^{p-4}$.

From our analysis above we expect that at large distances the antibranes are attracted to the O^{+}-plane (which has the same mass and opposite charges) and are repelled by the O^{-}-plane.

The gauge group associated with N antibranes plus an orientifold plane is USp(N) for the O^{+}-plane, and SO(N) for the O^{-}-plane. Scalars parametrizing the positions of the antibranes transform in symmetric (antisymmetric) representations if the gauge group is USp(N) (SO(N)). The fermions transform the other way around, being in the antisymmetric representation for the O^{+}-plane and in the symmetric representation for O^{-}-plane.

The Möbius strip amplitude. Let us first consider the case where the branes and orientifolds are attracted to one another: Op^{+} planes and Dp-antibranes. We compute their interaction by evaluating the disk and cross-cap amplitudes given the tension and Ramond-Ramond charges of the antibranes and orientifold planes respectively. The contribution to the vacuum energy of each of these amplitudes is:

$$\mathcal{D} = \bar{N} T_{Dp} V_p \tag{4.1}$$

$$\mathcal{C} = 2^{p-4} T_{Dp} V_p \,, \tag{4.2}$$

where $\overline{N}$ is the number of anti-Dp-branes, $T_{Dp} = [g_s (2\pi)^p l_s^{p+1}]^{-1}$ is the tension of a Dp-brane and V_p is the volume of the dimensions parallel to our objects (for convenience

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we consider these dimensions to be toroidally compactified). Both objects have positive tension so the total tension at this order is,

$$\mathcal{C} + \mathcal{D} = (\bar{N} + 2^{p-4}) T_{Dp} V_p \,. \tag{4.3}$$

The next order correction (one loop in open string counting) comes from the cylinder, the Möbius strip and the Klein bottle [18]. The cylinder and the Klein bottle amplitudes are zero, because a nonzero result requires both a boundary and a cross-cap amplitude since it requires the minimum combination which breaks all of the bulk supersymmetry. This leaves the Möbius strip as the only nonzero amplitude:

$$\mathcal{M} = -\frac{\overline{N} V_p}{l_s^{p+1}} \int_0^\infty \frac{dt}{t} \frac{e^{-2r^2 t/(\pi l_s^2)}}{2(8\pi t)^{(p+1)/2}} F(q^2) \,, \tag{4.4}$$

where r is the separation between the antibrane and the orientifold plane and $F(q^2)$ is the contribution from the string oscillator modes:

$$F(q^2) = \left(\frac{f_2(q^2) f_4(q^2)}{f_1(q^2) f_3(q^2)}\right)^8 = \frac{\theta_{10}^4(0|-it)\theta_{01}^4(0|-it)}{\eta^{12}(-it)\theta_{00}^4(0|-it)} \,. \tag{4.5}$$

Here $q = e^{-\pi t}$ and F has the asymptotics $\lim_{t\to\infty} F(q^2) = 16$ and $\lim_{t\to 0} F(q^2) = 256 t^4$.

Large separations between the branes and orientifold plane corresponds to the UV in the open string picture and IR in the closed string picture. In this limit the main contribution arises from the exchange of massless closed-string modes: the graviton, dilaton and Ramond-Ramond fields. This corresponds to the limit $t \to 0$ in the integral (large t contribution are suppressed in the integral). The contribution of the oscillator modes reduces to $F(q^2) \to 2^8 t^4$ and the integral becomes

$$\mathcal{M} = -\frac{2^8 \overline{N}\, V_p}{l_s^{p+1}} \int_0^\infty \frac{dt}{t} \frac{t^4 e^{-2r^2 t/(\pi l_s^2)}}{2(8\pi t)^{(p+1)/2}} = -2^{p-4} \overline{N} g_s T_p V_p \left[2^8 \pi \Gamma\left(\frac{7-p}{2}\right)\right] \left(\frac{l_s}{r}\right)^{7-p} \,. \tag{4.6}$$

This is the expected result: the power-law associated with the exchange of massless closed-string states. In this limit the result can be obtained by the supergravity approximation in the weak-field approximation, and is the same as is obtained in the branonium case.

The difference with the branonium case comes at short distance. For branonium an open string tachyon appears at distances of the order of the string scale, signaling the decay of the antibrane with one of the branes in the stack of source branes. In the orientifold case there is no tachyon at short distance, so the system is expected to be stable. At short distance the best description is found in the open-string picture, and at the massless level we find a field theory description in $p+1$ dimensions consisting of a $\mathrm{USp}(\overline{N})$ gauge group with $9-p$ scalars in the adjoint representation (i.e. the fields parameterizing the position of the branes) and 2^{9-p} fermions of both chiralities (in even dimensions).

To obtain the potential for the adjoint scalars in this regime we must go to the $t \to \infty$ limit. There $F(q^2) \to 2^4$ and the potential takes the form:

$$\mathcal{M} = g_s V_p T_p \sum_{n=0} a_n \left(\frac{r}{l_s}\right)^{2n} \,, \tag{4.7}$$

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where

$$a_n = (-1)^{n+1}\overline{N}k_{n-(p+3)/2}\left(\frac{2^{n-(p+5)/2}\pi^{(p-n-1)/2}}{n!}\right), \tag{4.8}$$

and

$$k_j = \int dt\, t^j F(q^2)\,, \tag{4.9}$$

is a positive number if $-5 < j < -1$. Numerically, $k_{-9/2} = 21.75$, $k_{-7/2} = 13.47$, $k_{-5/2} = 13.92$, $k_{-4} = 13.82$, $k_{-3} = 12.31$.

From these expressions there are two results which bear emphasis. First, the a_0 term describes a correction to the vacuum energy, which is finite and negative for $p < 7$. We can see that the one-loop correction is therefore acts to reduce the tree-level tension. Second, the a_1 term gives a harmonic-oscillator, r^2, potential near $r = 0$ and so represents a positive mass for the adjoint fields representing the Dp-brane center-of-mass motion.

4.2 Interaction potential

The potential which results from these considerations is drawn in figure 5. A useful interpolation which captures both the $1/r^{7-p}$ behaviour at long distances and the r^2 behaviour at short distance is the lorentzian

$$V(r) = -\frac{g_s T_p\,|a_0|}{(1+\alpha\, r^2)^{(7-p)/2}} \tag{4.10}$$

where α is a constant that can be adjusted to match the long-distance behaviour.

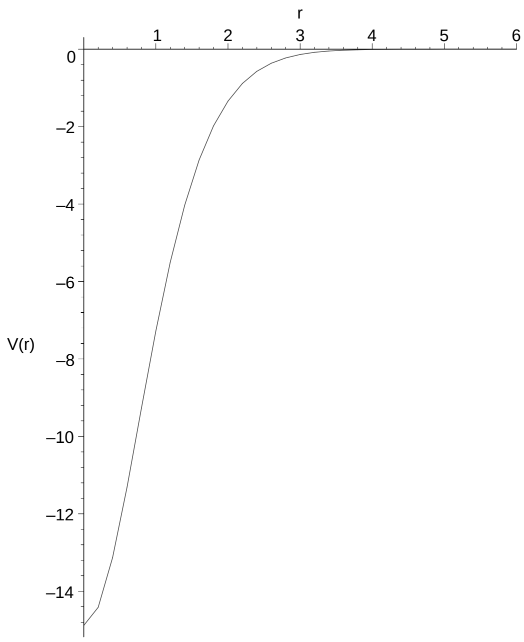

Figure 5: Attractive potential for an orientifold plane Op^+ and an antiD-brane from the Möbius strip. At long distances the result coincides with a brane-antibrane system (potential of the form $1/r^n$). At short distances the system is better understood in the open string picture where the scalar fields representing the position along the transverse dimensions take a mass.

Long-distance regime. The analysis in this regime follows the same lines as for branonium. This is because, from the supergravity point of view, the metric, dilaton and Ramond-Ramond forms cannot distinguish between an orientifold plane and a set of 2^{p-4} branes. Notice however that the limit of a large number of source D-branes (which is possible for branonium) cannot be reproduced here. In particular, for $p = 5$ the orientifold has the same tension and charge as a D-brane (taking into account the orientifold image). Because of this we cannot access the regime where nonlinear supergravity effects are important without leaving the regime of validity of our approximations.

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Short-distance regime. As explained earlier, the one-loop contribution gives a short-distance potential of the form $a_1 r^2$, with $a_1 \sim g_s/l_s^2$.

The one-loop vacuum amplitude is negative and so reduces the tree level tension, giving

$$\Lambda = [(\bar{N} + 2^{p-4}) + g_s a_0] T_{Dp} V_p \,. \tag{4.11}$$

We can see the limits of validity of the loop expansion by asking for the value of the string coupling for which the tree-level and one-loop contributions cancel is given by

$$g_s = \frac{(\bar{N} + 2^{p-4})}{|a_0|} \,. \tag{4.12}$$

Since $a_0 \sim -\overline{N}$, the transition between the two regimes occurs at $g_s \sim 1$.

4.3 Strong coupling regime

Ref. [10] makes several conjectures concerning the behaviour of this type of system at strong coupling. Let us consider the case of a $O3^+$-plane and an antibrane (2 counted in the covering space). The total Ramond-Ramond charge of the system is $q_{RR} = 1/2 - 2 = -3/2$. The gauge group at low energies is USp(2) = SU(2), with some fermions which are singlets (because they transform in the antisymmetric representation). There are also 6 scalars parameterizing the positions of the antibranes, which transform in the adjoint representation.

This system is conjectured to be S-dual (the coupling in the S-dual system is $\tilde{g}_s = 1/g_s$, and the string length is $\tilde{l}_s = g_s^{1/2} l_s$ to keep the Plank mass fixed) to an $O3^-$-plane containing one embedded anti-D3-brane. The Ramond-Ramond charge of this system is $q_{RR} = -1/2 - 1 = -3/2$. The world-volume theory has no gauge bosons and scalars (recall the antibrane is stuck at the position of the orientifold plane), but there is a fermionic zero mode. As is explained in [10] this conjectured duality illustrates two mechanisms of keeping D-branes close to orientifold planes: either by loop effects through the correction to the masses of the center-of-mass scalars, or by projecting out by the orientifold.

In the limit of very strong coupling the vacuum energy comes from the tension of the objects with $T \sim 1/\tilde{g}_s$,

$$\Lambda(g_s \to \infty) = \left[-\frac{1}{2} + 1\right] g_s \frac{V_p}{(2\pi)^3 l_s^4 g_s^2} \,. \tag{4.13}$$

The first-order correction to this result comes from the Möbius strip. This gives a constant contribution (the tree level scalars are projected out) that is the same as the a_0 of the S-dual case, but with opposite sign. So the sum of the tree-level and one-loop results at strong coupling is

$$\Lambda(g_s \to \infty) = \left(\left[-\frac{1}{2} + 1\right] g_s + |a_0|\right) \frac{V_p}{(2\pi)^3 l_s^4 g_s^2} \,. \tag{4.14}$$

By contrast, at small coupling the vacuum energy goes like

$$\Lambda(g_s \to 0) = \left(\left[\frac{1}{2} + 2\right] \frac{1}{g_s} - |a_0|\right) \frac{V_p}{(2\pi)^3 l_s^4} \,. \tag{4.15}$$

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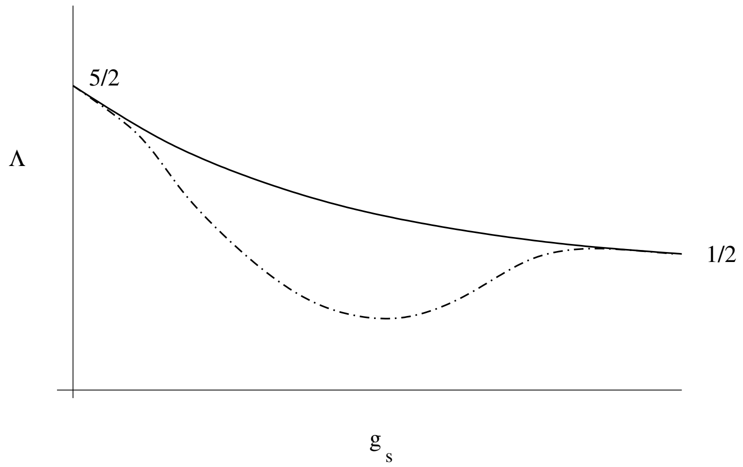

Figure 6: Schematic representation of the vacuum energy for the $O3^+$-antibrane system depending on the coupling. The two lines represent two different possibilities: in the solid line case the system is driven to string coupling while in the dashed line case the dilaton is stabilized.

In order to obtain the potential for the dilaton in the Einstein frame we should multiply these results by g_s, so at weak coupling we find

$$\Lambda(g_s \to 0) = \left(\left[\frac{1}{2} + 2 \right] - |a_0| g_s \right) \frac{V_p}{(2\pi)^3 l_s^4} , \tag{4.16}$$

while at strong coupling we have

$$\Lambda(g_s \to \infty) = \left(\left[-\frac{1}{2} + 1 \right] g_s + |a_0| \right) \frac{V_p}{(2\pi)^3 l_s^4 g_s} . \tag{4.17}$$

These results are what one might expect. If loop effects are ignored there is no potential for the dilaton, because it is the loop effects which know about the breaking of supersymmetry. Consequently the potential is constant. The fact that the vacuum energy at strong and weak coupling are not equal to one another also has to do with supersymmetry breaking. If we take a supersymmetric combination of D3-branes and orientifold 3-planes the potential is the same and flat.

We plot in figure 6 the schematic shape of the vacuum energy as a function of the coupling. It is not clear if there is a value for the coupling for which the vacuum energy can be zero or negative. It is also unclear if the dilaton is stabilized at any finite value (i.e. as would be the case for the dashed line in the figure) or not (in which case the D-brane is driven onto the $O3^-$ plane system). Notice that once $g_s > 5/(2|a_0|)$ the weak-coupling expression for Λ falls below the asymptotic value of the strong-coupling expression at infinity, and this would give evidence for the existence of a minimum to the extent that this coupling is still within the domain of applicability of leading-order perturbation theory.

Another interesting observation is that taking T-dualities of the Sugimoto model [11] and moving branes we can obtain a system having 16 $O3^+$-planes with 2 anti D3-branes attached to them, plus 48 $O3^+$-planes without branes. The total tension of this system is $64/g_s$ in D3-brane units. By applying the S-duality rules above one can get a system without dynamical D-branes, consisting of 16 $O3^-$-planes with 1 anti-D3-brane (that

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cannot move from it) and 48 $O3^-$-planes with a non-dynamical D-brane on top of each one. The total tension of this dual system is $32g_s$. The same considerations regarding the stabilization of the coupling that were made in the non-compact case also apply here.

4.4 Repulsive case

We now consider the case with an orientifold plane Op^- and an antibrane. The arguments proceed exactly as in the previous example, with the following differences:

- The open-string description implies an SO(N) gauge group and scalars in the adjoint (with fermions in symmetric) representations,
- The tension of the orientifold has an opposite sign (negative tension), while the antibranes carry positive tension. This means that the disk plus cross-cap amplitudes in this case are
$$\mathcal{C} = (\bar{N} - 2^{p-4})T_{Dp}V_p \,. \tag{4.18}$$
- The Möbius amplitude gives a repulsive potential (the same shape as before, but with a global minus sign). The antibranes are then unstable if placed on top of the orientifold and the system becomes unstable at one loop.

5. Conclusions

With this paper we begin a discussion of the motion of various kinds of probe Dp'-(anti)branes about a stack of Dp source branes where $p' < p$, or orientifold planes. This study is a natural extension of the branonium (Dp-antibrane orbiting a stack of Dp-branes) analysis, of ref. [1], and the remarkable existence we find here of simple integrable relativistic motion generalizes the same property which was found there. We call the bound states, when these exist, 'branic' or 'oriental' atoms (depending on whether it is a source brane or orientifold plane about which the orbits occur) to underline that unlike branonium these systems are stable against mutual annihilation.

For the Dp'-Dp-brane systems we examine the relativistic equations of motion, and provide a solution by quadratures for the fully relativistic orbits. The long-range force experienced by the probe brane is attractive when $p - p' < 4$, is flat in the BPS case where $p - p' = 4$ and is repulsive when $p - p' > 4$. These three conditions also coincide with the conditions for where the lowest level of the open-string sector of the theory is tachyonic, massless or massive respectively. These trajectories can involve bound states having Keplerian conic sections as orbits if $p = 6$ and $p' = 4, 2$.

We examine the case of probe branes moving around D6 source branes in a bit more detail, and in particular consider the perspective which obtains if these systems are lifted to M-theory in 11 dimensions. There are no D6-branes in M-theory, but the fields they source in the IIA theory survive (like the Cheshire Cat's smile) into 11 dimensions as a Taub-NUT-type metric configuration, and we show explicitly how the orbits of probe branes in this Taub-NUT space duplicate the motion of probes about a D6 brane in IIA supergravity.

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Finally, we consider some aspects of the interaction potential experienced by an anti-brane/orientifold plane system, explicitly exhibiting the cross-over from the long-distance, weak-field regime to the short-distance, harmonic-oscillator regime.

It is interesting to speculate about the possible physical implications of these systems, especially for early universe cosmology. Understanding the interactions and relative motion of D-branes may play an important role in scenarios of brane dynamics in the early universe, such as brane gases [12], D-brane inflation [13] or alternatives [14], mirage cosmology [15], *etc.* For this a more complete analysis of, at least, the most interesting systems that we studied here (such as D6-D2 and D-brane-orientifold) might be needed.

Some of the analyses done for the branonium case can in principle also be generalised to these systems, such as to investigate stability against perturbations in the transverse direction of the probe brane, radiation into bulk and brane degrees of freedom, orbital decay time, and so on. In particular, an understanding of the behaviour of these systems after compactification is very likely required for real applications.

It is striking that the $p - p' = 2, 4$ cases have the same classical trajectories as equivalent non-relativistic systems, even though the original systems are fully relativistic. A similar situation also arose in the branonium system. For BPS systems the potential of the equivalent non-relativistic lagrangian encodes the velocity-dependent forces of the original relativistic problem. For non-BPS systems the equivalent non-relativistic potential has the same functional form as the full system's potential in the non-relativistic limit, although this is not true for the BPS case. It is not clear why these systems are integrable, and whether it is only the BPS cases for which the equivalent and non-relativistic potentials differ. In the presence of the Wess-Zumino term — i.e. when $p = 6$ and $p' = 0$ — the equivalent system was the lagrangian for a charge moving in the field of a dyon. A further understanding of these relations is certainly needed.

Finally we may consider yet more complicated systems. The relative motion of an anti-brane in a lattice of Dp-branes (which one might call the 'brane transistor') could have interesting properties and possible applications in cosmology. Also systems of intersecting branes, which are known to have interesting phenomenological and cosmological properties, have similar background geometries as those studied here and the discussion presented here may be generalizable to these cases. There is no shortage of directions deserving closer scrutiny.

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